day 44

3.8/74 Surface area of a cone.

$$A = \pi r \sqrt{r^2+h^2}$$

multiply... product rule

find $\frac{dr}{dh}$ so... implicit differentiation w.r.t. h

$(r^2+h^2)^{1/2}$

$$\frac{dA}{dh} = \pi\left[\frac{dr}{dh}\sqrt{r^2+h^2} + r\frac{d}{dh}\left(\sqrt{r^2+h^2}\right)\right]$$

$$\frac{dA}{dh} = \pi\left[\frac{dr}{dh}\sqrt{r^2+h^2} + r\left(\frac{1}{2}(r^2+h^2)^{-\frac{1}{2}}\cdot\left(2r\frac{dr}{dh}+2h\right)\right)\right]$$

$$\frac{dA}{dh} = \pi\sqrt{r^2+h^2}\,\frac{dr}{dh} + \frac{r^2}{\sqrt{r^2+h^2}}\frac{dr}{dh} + \frac{hr}{\sqrt{r^2+h^2}}$$

$$\left(\pi\sqrt{r^2+h^2} + \frac{r^2}{\sqrt{r^2+h^2}}\right)\frac{dr}{dh} = \frac{dA}{dh} - \frac{hr}{\sqrt{r^2+h^2}}$$

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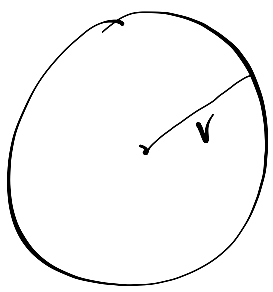
3.11/11

day 44



initially, $r = 50'$

$\frac{dr}{dt} = -2$ ft/min

when $r = 10$ft, what is $\frac{dA}{dt}$?

2: $A = \pi r^2$

$$\frac{d}{dt}(\pi r^2) = \pi \frac{d}{dt}(r^2) = \pi\left(2r\frac{dr}{dt}\right)$$

3: $\frac{dA}{dt} = 2\pi \underbrace{r}_{10} \underbrace{\frac{dr}{dt}}_{-2}$

4: $\frac{dA}{dt} = 2\pi(10)(-2) = -40\pi \ ft^2/min$

3.10/22

day 44

chain rule

$$\frac{d}{dt}\left(\ln(f(t))\right) = \frac{1}{f(t)} \cdot f'(t)$$

$$f(t) = \ln\left(\tan^{-1}(t)\right)$$

$$f'(t) = \frac{1}{\tan^{-1}(t)} \cdot \frac{d}{dt}\left(\tan^{-1}(t)\right)$$

$$f'(t) = \left(\frac{1}{\tan^{-1}(t)}\right)\left(\frac{1}{1+t^2}\right)$$

$$\tan^{-1}(t) = \arctan(t)$$

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3.10/47)

day 44

Find $(f^{-1})'(3)$ if $f(x)=x^3+x+1$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))}$$

$3=x^3+x+1$

so $f'(x)=3x^2+1$
$f(1)=3$

so

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{(3x^2+1)}\Bigg|_{x=f^{-1}(3)}$$

$$= \frac{1}{(3x^2+1)}\Bigg|_{x=1} = \frac{1}{4}$$

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day 44

3.10/43) $f(x)=\sqrt{x}$; $(2,4)$ find the slope of the line tangent ... to f^{-1}

$$(f^{-1})'(x)=\frac{1}{f'(f^{-1}(x))}$$

so $$f^{-1\prime}(2)=\frac{1}{f'(f^{-1}(2))}=\frac{1}{f'(4)}=\left.\frac{1}{\frac{1}{2\sqrt{x}}}\right|_{x=4}=4$$

$$\left[\frac{d}{dx}(\sqrt{x})=\frac{d}{dx}(x^{\frac{1}{2}})=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}\right]$$

So ... $(y-4)=4(x-2)$

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4.1) maxima & minima

day 44

Singular maximum, minimum, extremum

(jumbo-est) (shrimpy-est) either maximum or minimum

plural (fr. Latin) maxima, minima, extrema

Common abbrev max min

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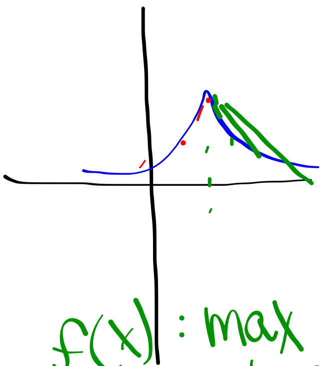
Point of 4.1 :

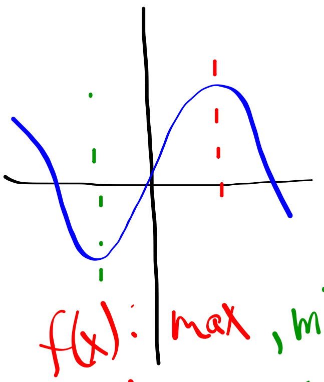
we can identify possible extrema by looking at values & derivative

Consider 3 graphs of $f(x), g(x), h(x) \ldots$



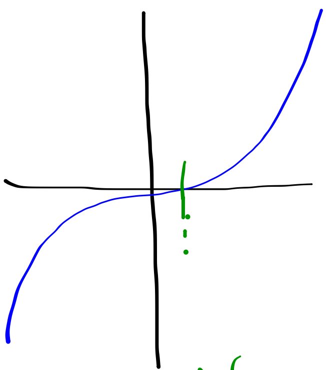
$f(x)$: max

$f'(x)$: not defined



$f(x)$: max , min

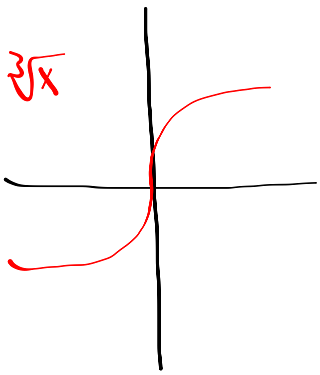
$f'(x)$: 0 , 0



$f(x)$: No max. or min

$f'(x) = 0$

$y = \sqrt[3]{x}$



$f(x)$: No max. or min

$f'(x)$: not defined

day 44

to go "backwards"

IMP idea

vocab: An x-value where

* $f(x)$ exists, continuous
* $f'(\text{x-value}) = 0$ OR $f'(x)$ dne at the x-value

is called a CRITICAL POINT.

"Rule": Extreme values of $f(x)$ occur at:

→ CRITICAL POINTS of f

or → endpoints of the domain.

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